Feature-Based Dynamic Matching

Akshit Kumar

Joint work with Yilun Chen, Yash Kanoria and Wenxin Zhang

Digital Matching Markets

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Home services platforms which provide on-demand services like cleaning, maintenance, etc.

handy

UC Urban Company

k: kandua

SweepSouth®

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- Customers arrive online and specify requests and service preferences (eg. location, time, price)

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- Have access to a pool of heterogeneous service providers which are differentiated by their features (eg. location, rating, hours of operation)
- Customers arrive online and specify requests and service preferences (eg. location, time, price)
- Customers need to be matched in near real time to a service provider

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Salient Facets of these platforms…

- Have access to a pool of heterogeneous service providers which are differentiated by their features (eg. location, rating, hours of operation)
- Customers arrive online and specify requests and service preferences (eg. location, time, price)
- Customers need to be matched immediately and irrevocably to a service provider

Key Operational Challenge

How should centralized matching platforms match customers arriving over time to maximize overall quality of matches generated?

In A Nutshell

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- We study dynamic matching in two-sided markets with heterogeneous demand and supply

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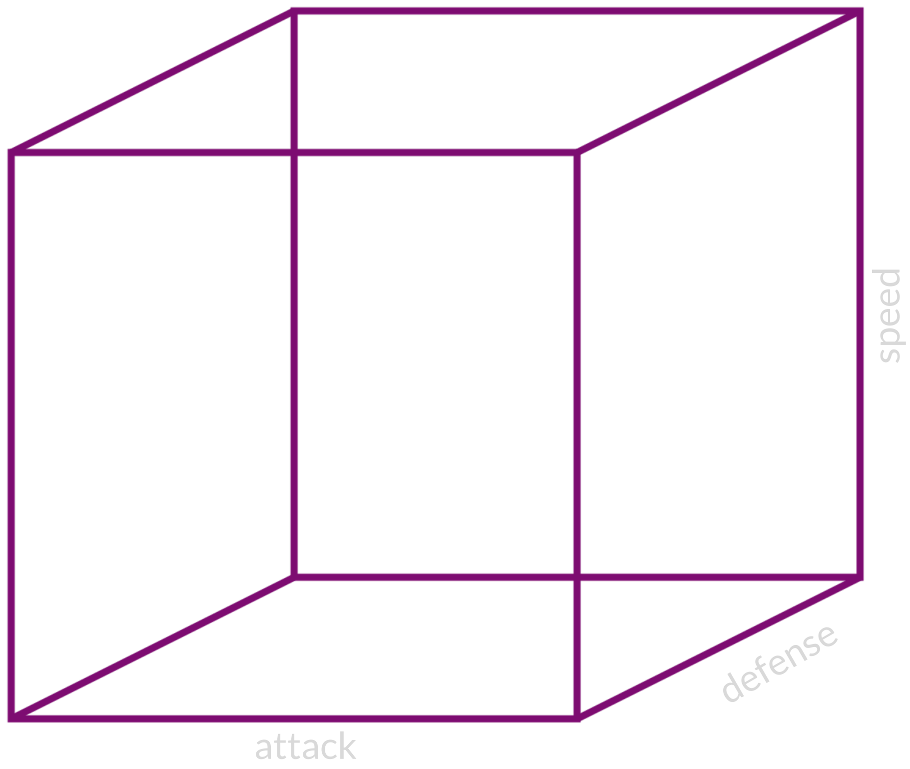
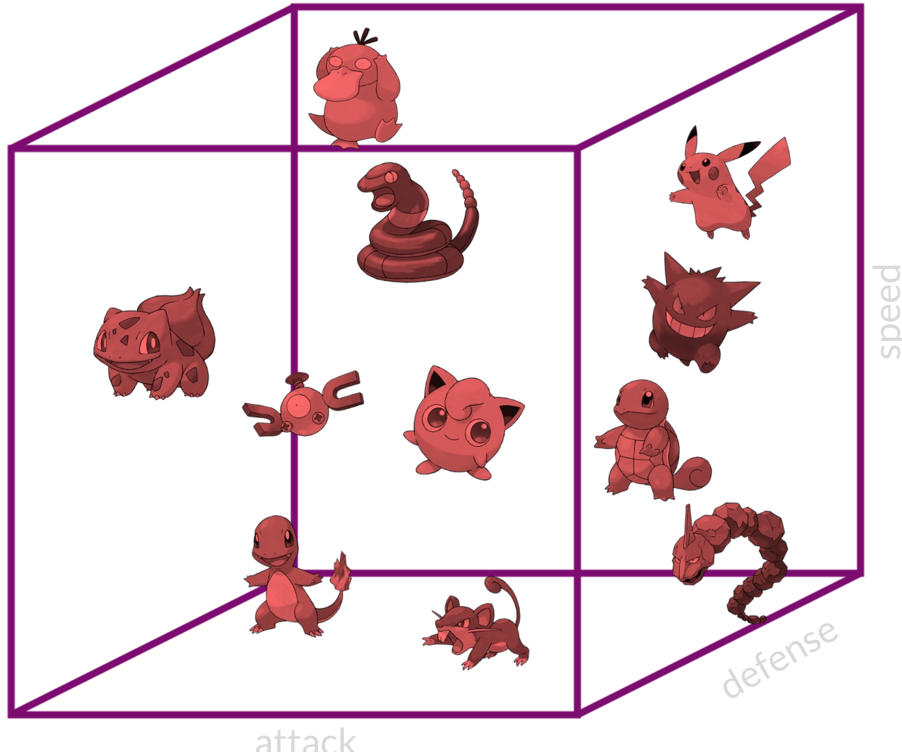
Model



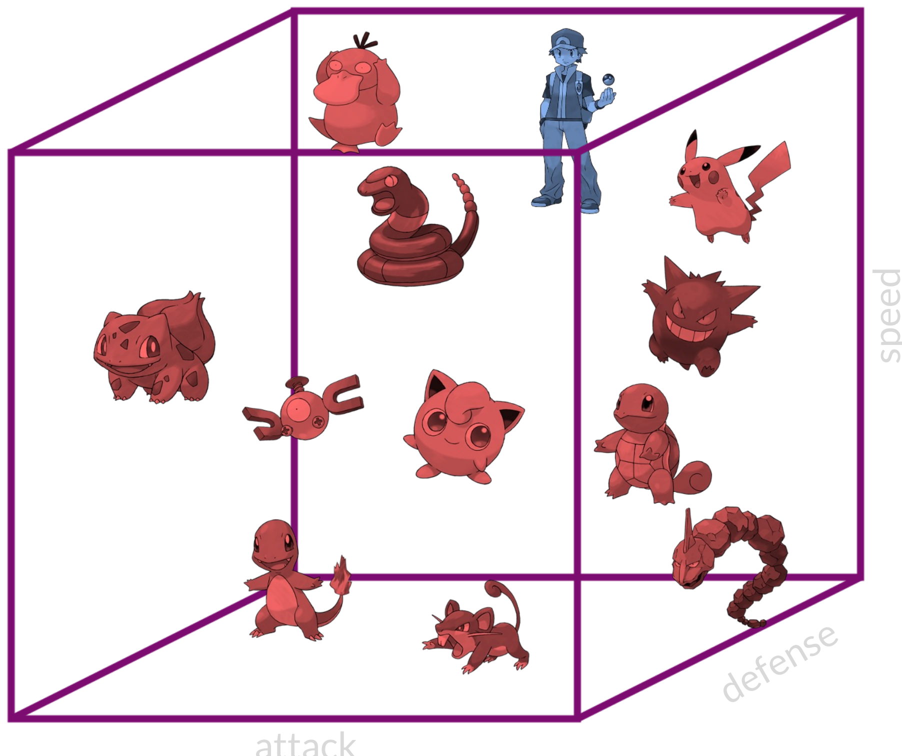
Supply is represented by a **feature** vector in a d-dimensional space

Model



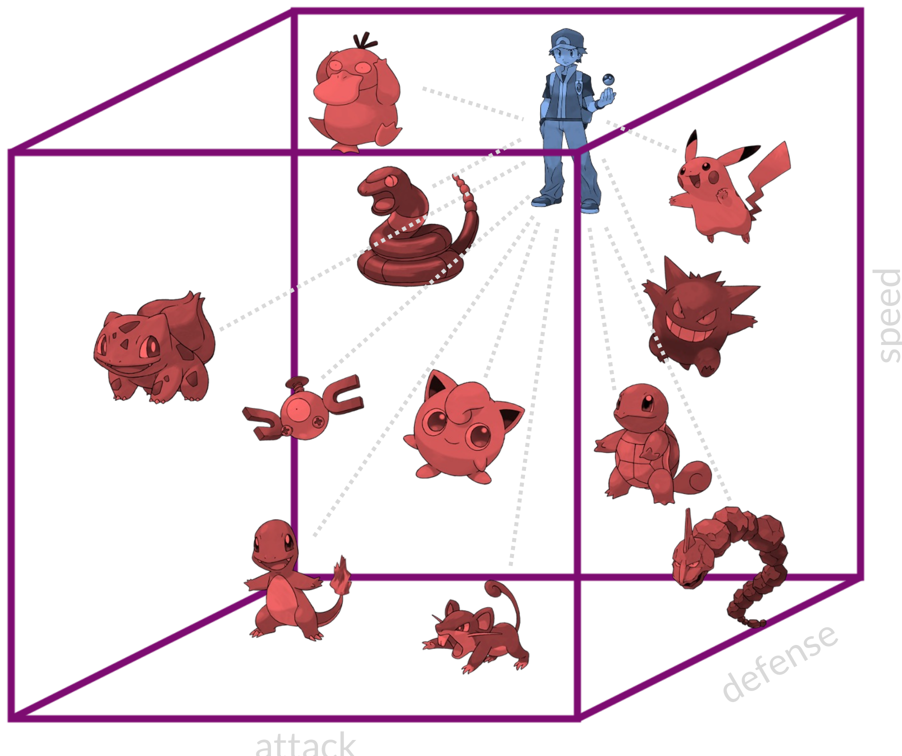
Platform has n supply units drawn i.i.d from distribution Q

Model



At time t, a demand unit with **weight** vector is drawn i.i.d from **known** P

Model



Platform must irrevocably match a demand unit to supply unit

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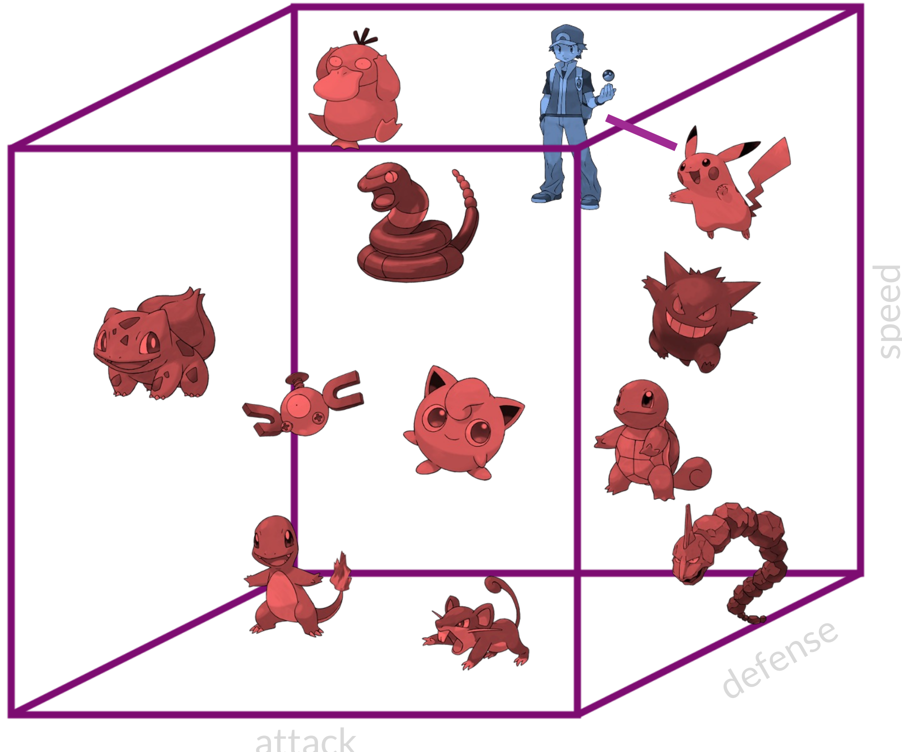

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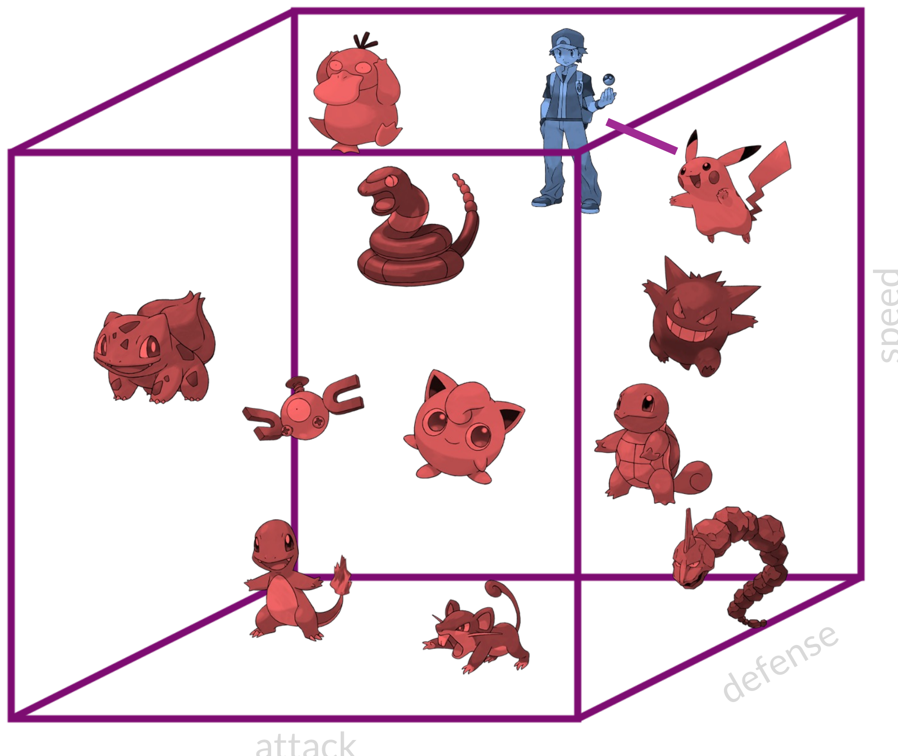
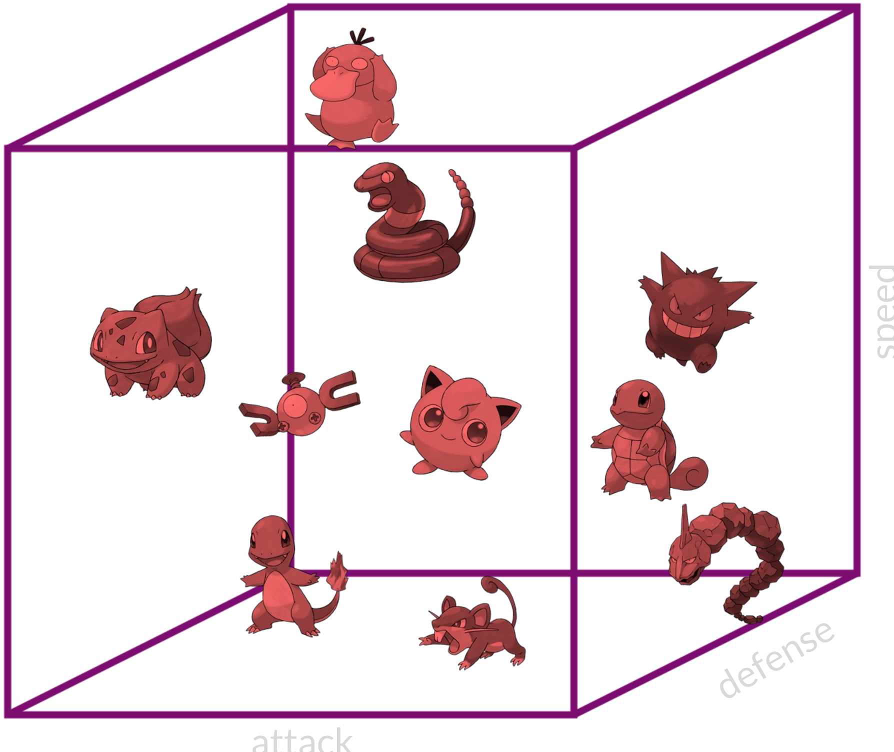


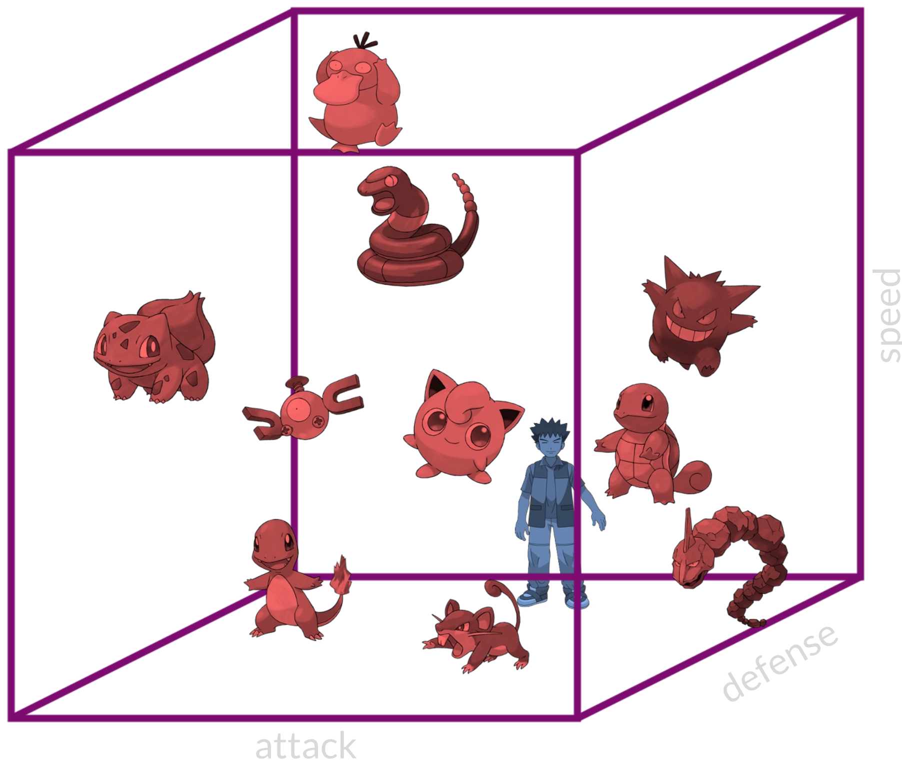
The matching quality is measured by the **dot product** ⟨  ,  ⟩

Model



Both demand and supply leave upon matching

Model



The process repeats for a total of n time steps

Objective

Platforms' Objective

maximize the expected average match quality

$$\max_{\pi} \frac{1}{n} \mathbb{E}[\textstyle\sum_{k=1}^{n} \langle X_k, Y_{\pi(k)} \rangle]$$

equivalently, minimize the **regret** with respect to the fluid benchmark

fluid benchmark is the optimal transport between the demand and supply distribution

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Algorithmic Desiderata

"simple" dynamic matching algorithms with $o(1)$ (vanishing) regret

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Dynamic two-sided matching with few types

Talluri & van Ryzin (2004); Vera & Banerjee (2021); Banerjee, Freund & Lykouris (2022)

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high dimensional features

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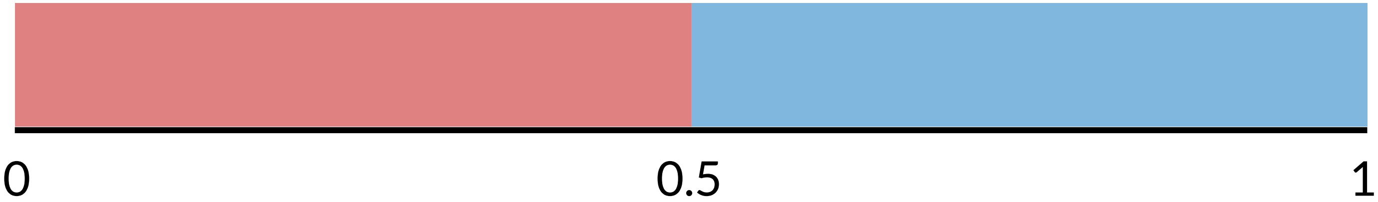
"… there is no greater disaster than greed" ~ Laozi



Supply distribution is uniform over [0,0.5]
Demand distribution is uniform over [0.5,1]

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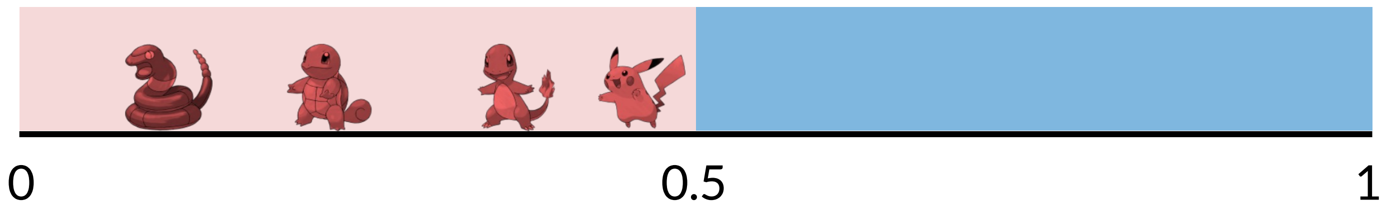

Platform has n supply units drawn i.i.d from Q

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Demand arrives, drawn i.i.d from distribution P

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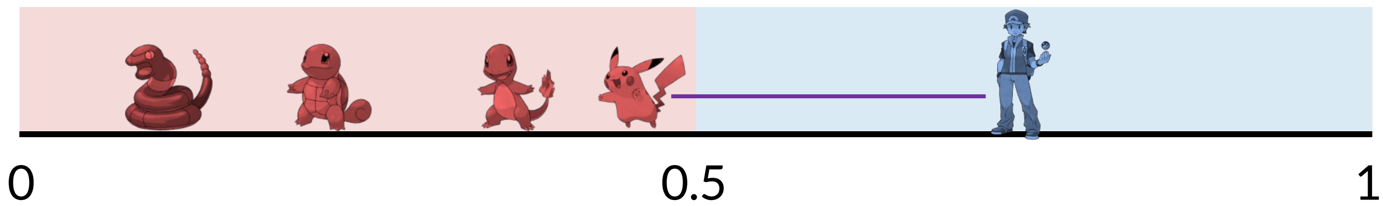

Greedy matches the demand unit to the **closest** supply unit

since it myopically maximizes the dot product

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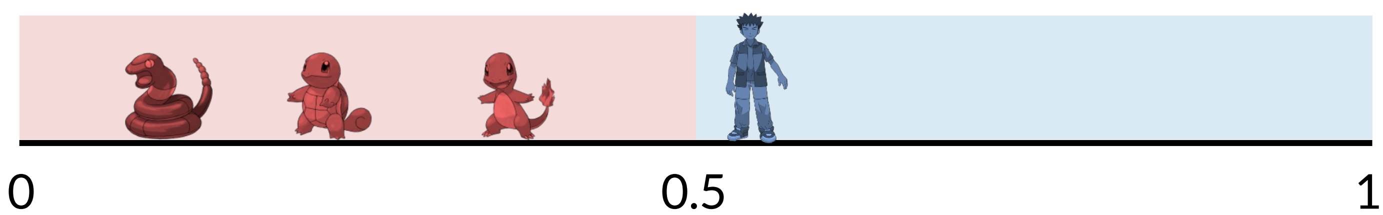

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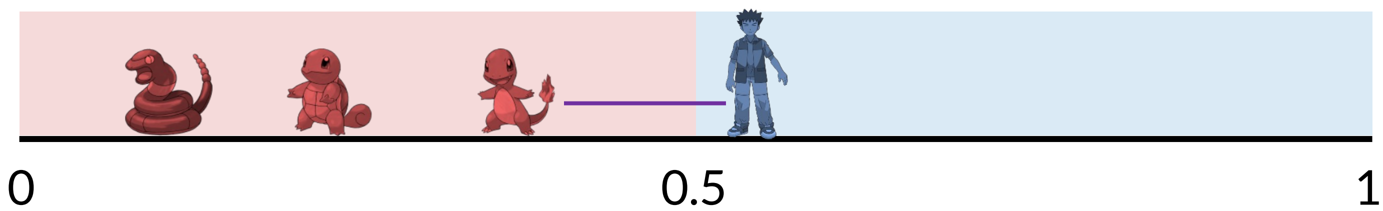

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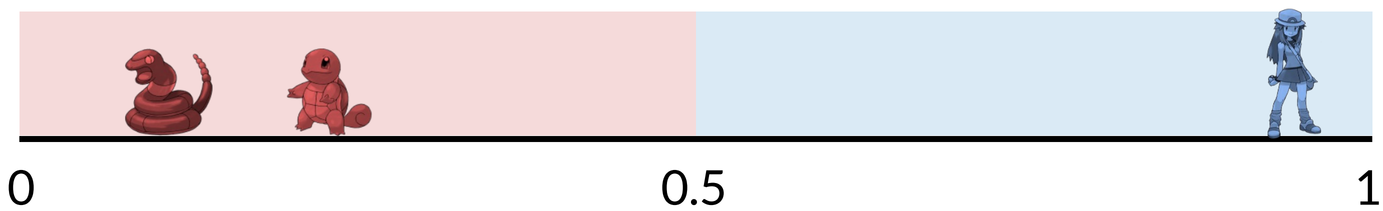

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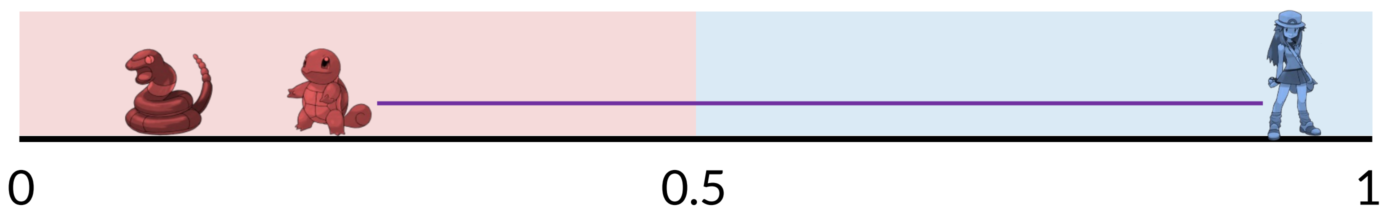

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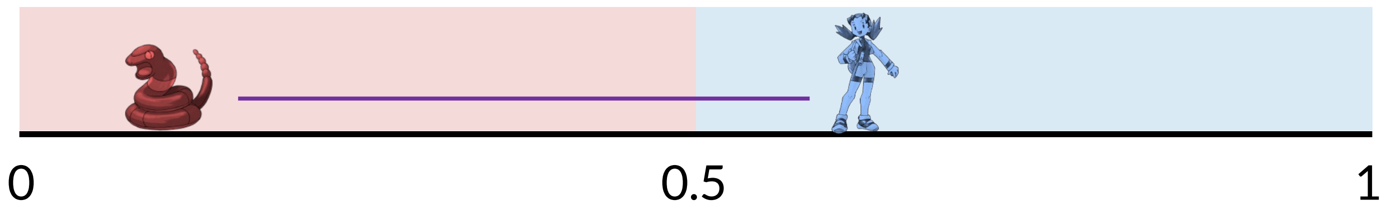

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Greedy produces a **random** matching between demand and supply

Expected average matching quality of Greedy is 3/16 for any n

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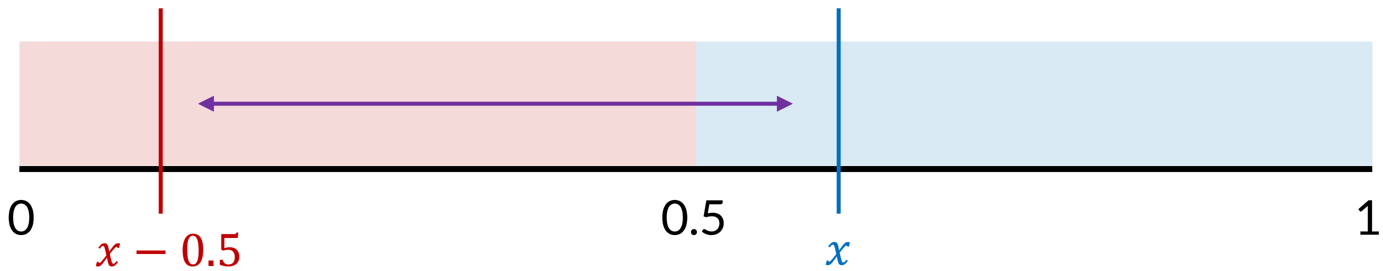

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In the fluid limit, demand unit x is matched to supply unit $x - 0.5$

Value of the fluid benchmark is $5/24$

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Greedy is not forward-looking and hence results in non-vanishing regret

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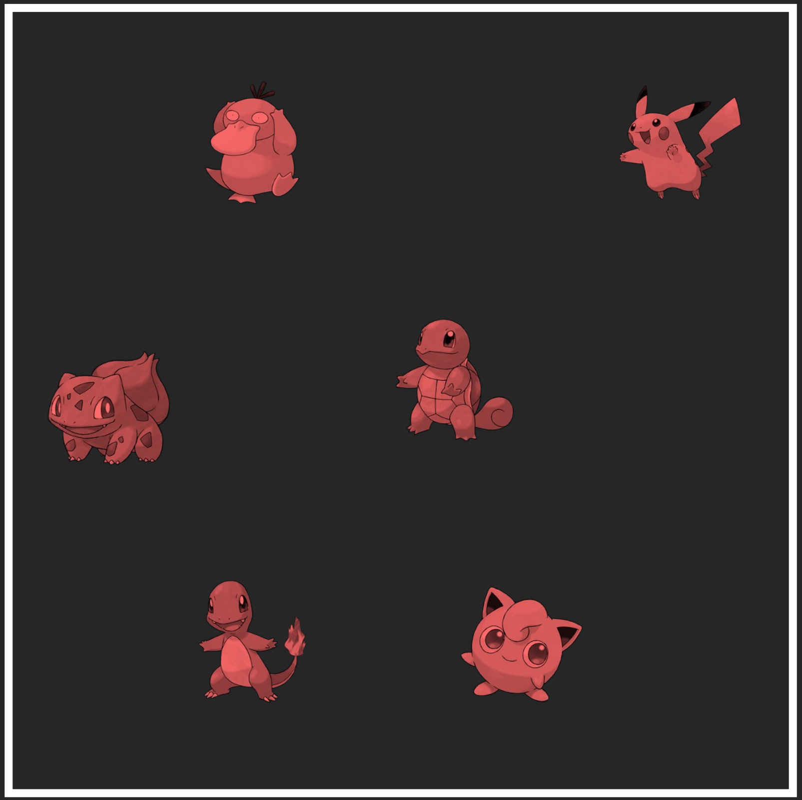

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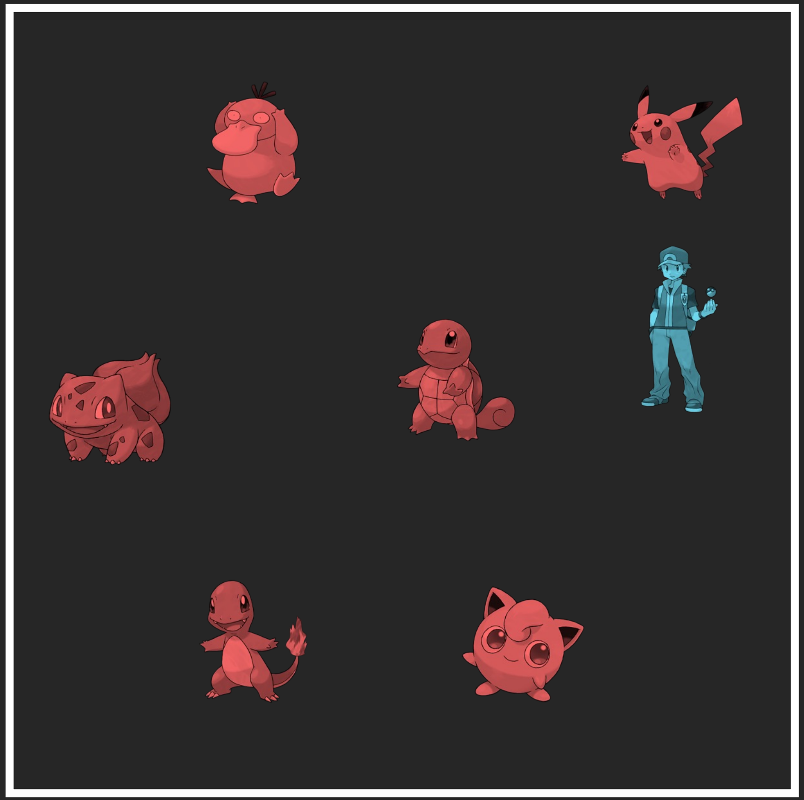
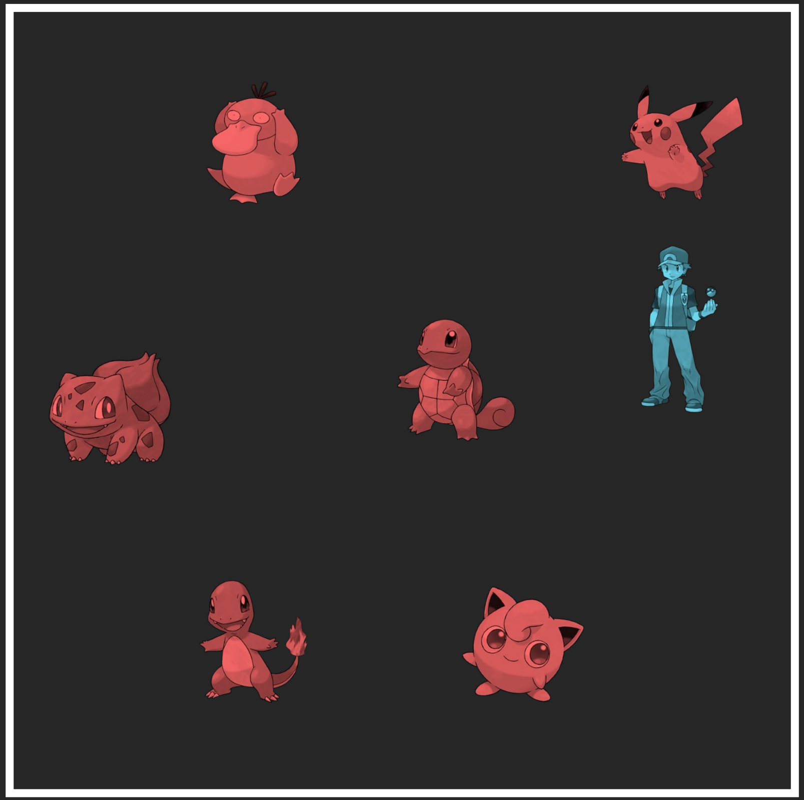


demand unit arrives

SOAR

Simulate

a future demand scenario



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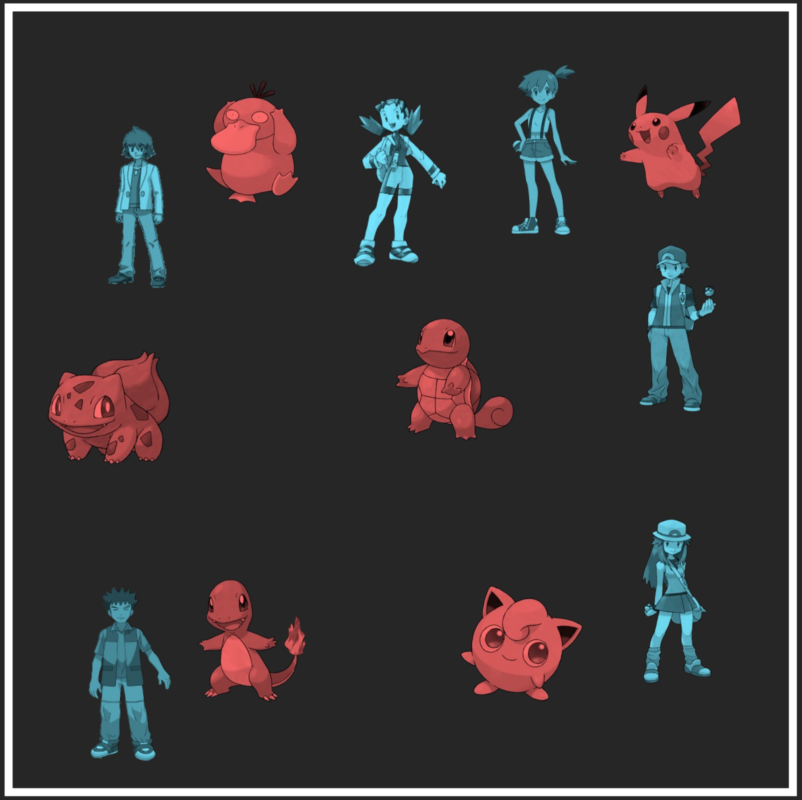

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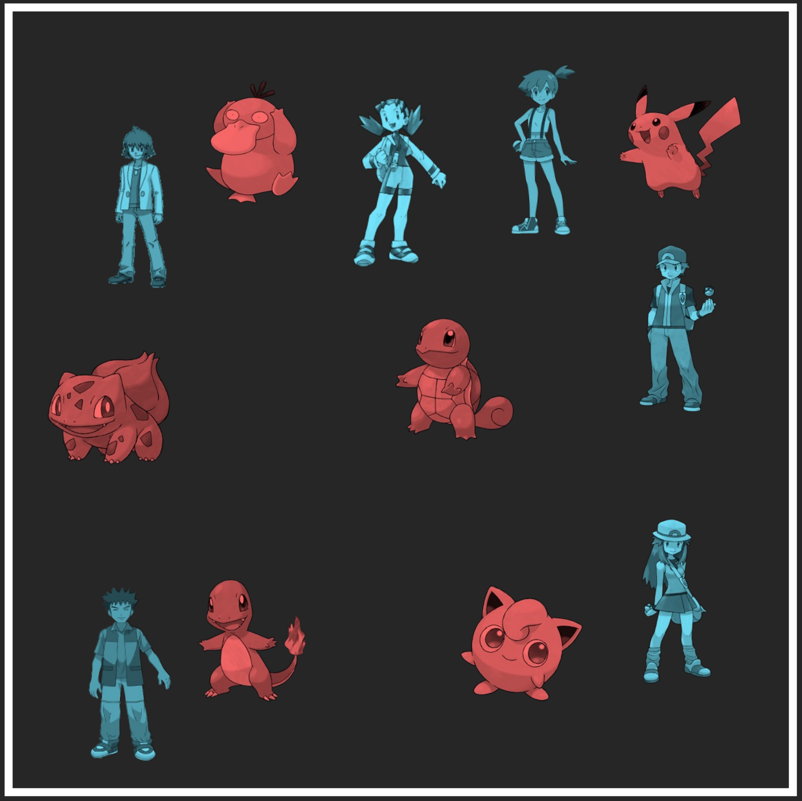

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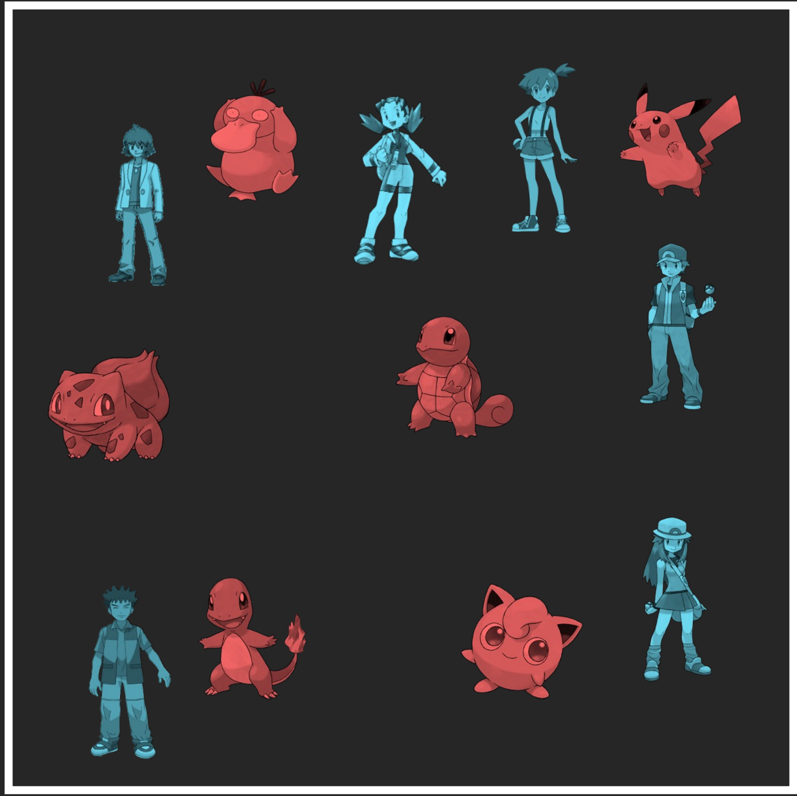

compute the optimal
weighted bipartite matching

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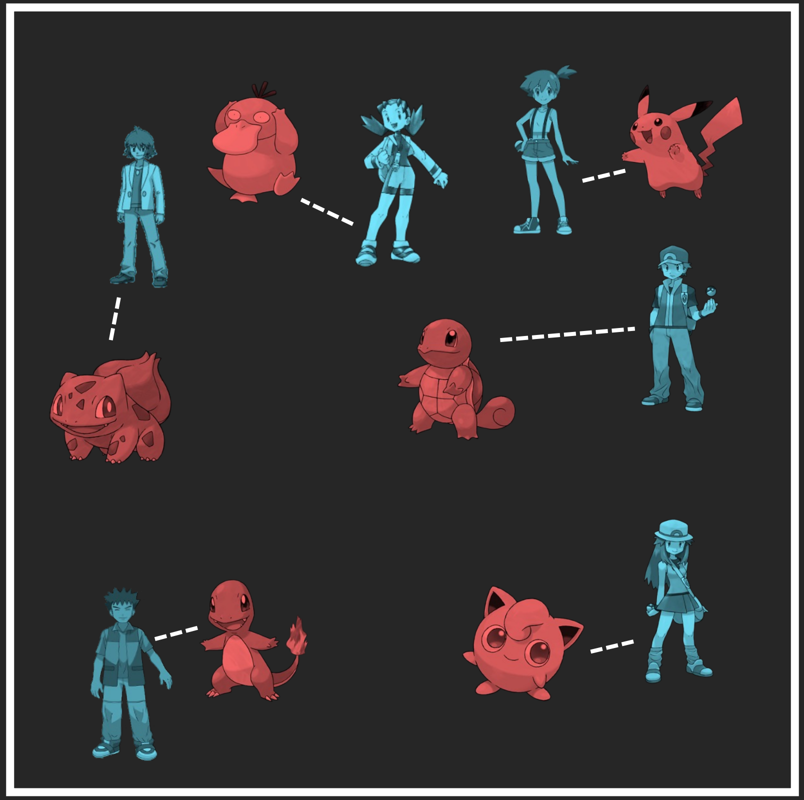

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Optimize

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Assign

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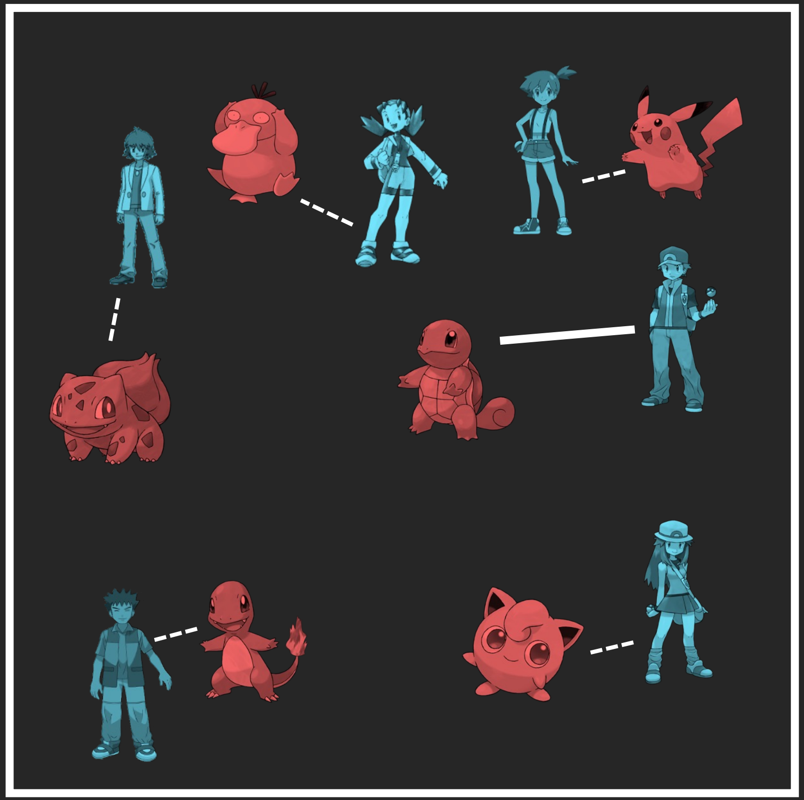

SOAR

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Assign

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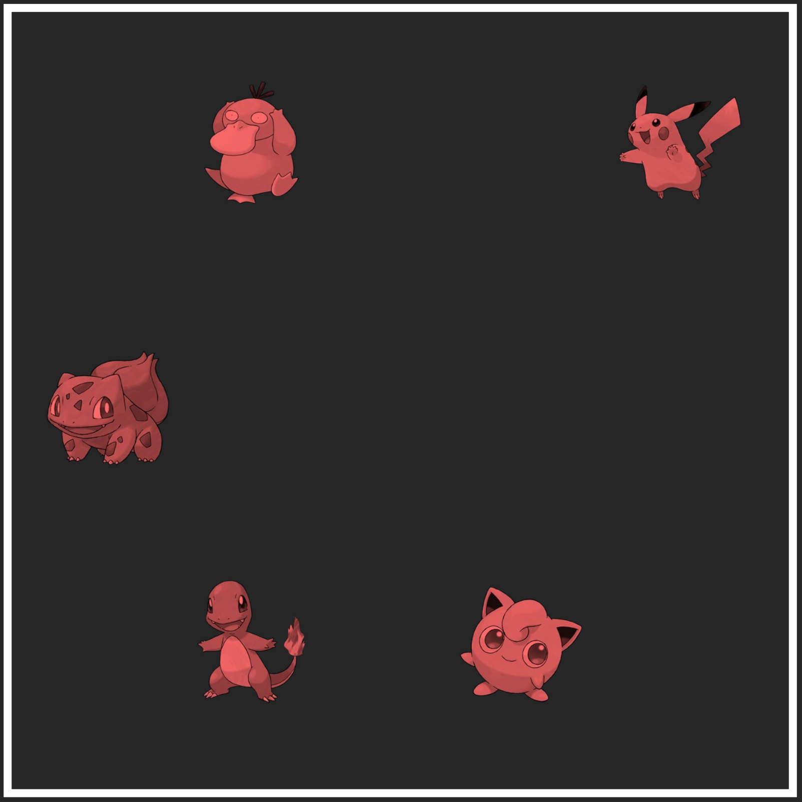
Repeat

with the remaining supply



Near-Optimality of SOAR

match quality function $\varphi(X, Y) = \langle X, Y \rangle$

demand distribution P and supply distribution Q

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	P, Q are smooth	P, Q are arbitrary
SOAR	$\tilde{O}\left(n^{-(\frac{2}{d} \wedge 1)}\right)$	$\tilde{O}\left(n^{-(\frac{2}{d} \wedge \frac{1}{2})}\right)$

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SOAR	$\tilde{O}\left(n^{-(\frac{2}{d} \wedge 1)}\right)$	$\tilde{O}\left(n^{-(\frac{2}{d} \wedge \frac{1}{2})}\right)$
Fundamental Limit	$\tilde{\Omega}\left(n^{-(\frac{2}{d} \wedge 1)}\right)$	$\tilde{\Omega}\left(n^{-(\frac{2}{d} \wedge \frac{1}{2})}\right)$

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$$U_n^{\mathrm{SOAR}} = \frac{1}{n} \sum_{k=1}^{n} U_k^{\mathrm{off}}$$

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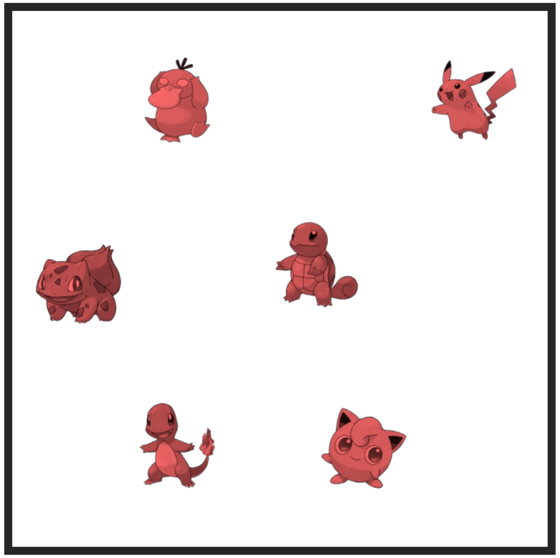
Given n i.i.d supply units, each supply unit is equally likely to be matched to the incoming demand (before observing the location of the demand unit)



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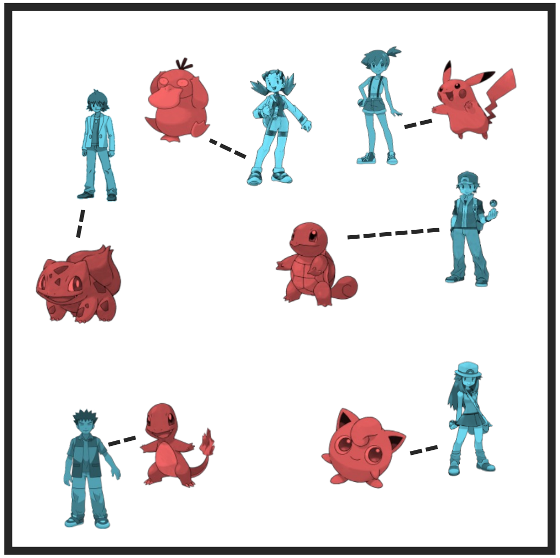

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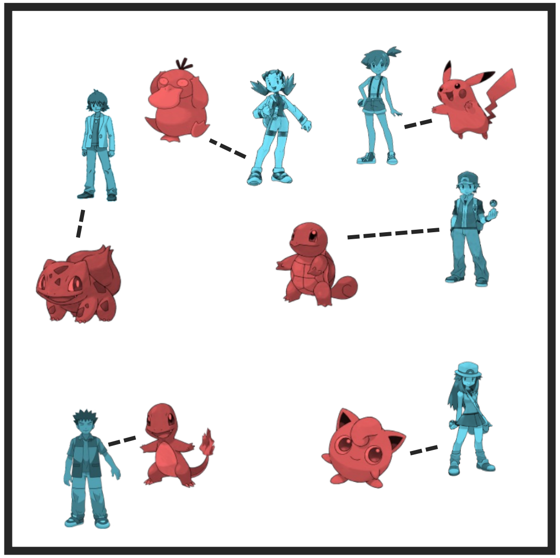
Expected matching quality of SOAR with n i.i.d supply units is U_n^{off}



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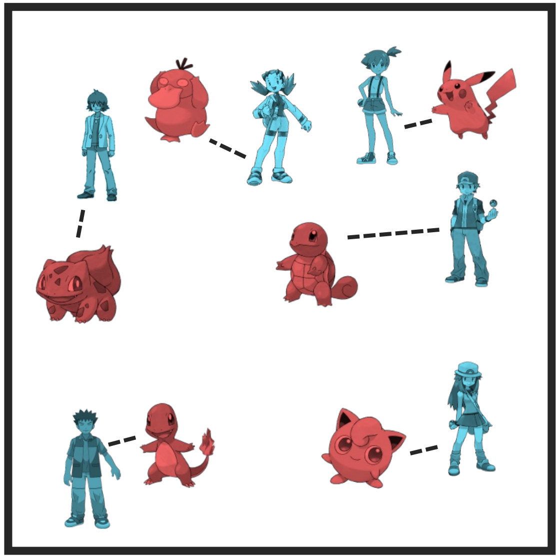
- Expected matching quality of SOAR with n i.i.d supply units is U_n^{off}
- The remaining $n-1$ supply units are i.i.d



Meta Theorem

Let U_k^{off} denote the expected average matching quality when matching k demand and supply units drawn i.i.d from P and Q respectively. Then the expected average matching quality under SOAR is given as

$$U_n^{\mathrm{SOAR}} = \frac{1}{n}\sum_{k=1}^{n} U_k^{\mathrm{off}}$$

Meta Theorem for Regret

Let U_k^{off} denote the expected average matching quality when matching k demand and supply units drawn i.i.d from P and Q respectively. Then the expected regret of SOAR is given as

$$\text{Regret(SOAR)} = U^{\text{fluid}} - U_n^{\text{SOAR}} = \frac{1}{n}\sum_{k=1}^{n}(U^{\text{fluid}} - U_k^{\text{off}})$$

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Well studied object in the empirical optimal transport literature

(e.g.; rate of convergence of empirical Wasserstein distance)

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	P, Q are smooth	P, Q are arbitrary
SOAR	$\tilde{O}\left(n^{-(\frac{2}{d}\wedge 1)}\right)$	$\tilde{O}\left(n^{-(\frac{2}{d}\wedge \frac{1}{2})}\right)$
Fundamental Limit	$\tilde{\Omega}\left(n^{-(\frac{2}{d}\wedge 1)}\right)$	$\tilde{\Omega}\left(n^{-(\frac{2}{d}\wedge \frac{1}{2})}\right)$

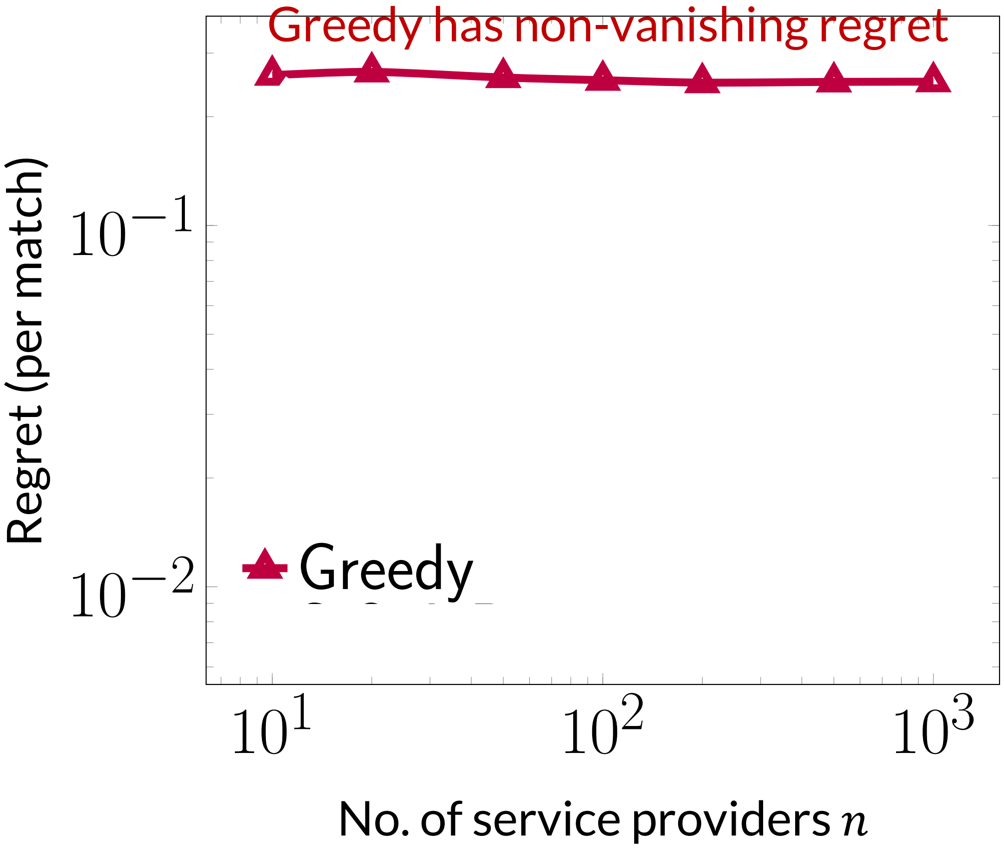
Extensions

- Our model can handle the case of scarce supply by utilizing dummy supply units
- Vanishing regret for dot product quality function with scarce supply and rejection cost
- Near-optimal guarantees for a general class of quality functions $\varphi(X,Y) = -||X-Y||^p$ (dot product is a special case with $p = 2$)

	P, Q are uniform	P, Q are arbitrary
SOAR	$\tilde{O}\left(n^{-(\frac{p}{d}\wedge 1)}\right)$	$\tilde{O}\left(n^{-(\frac{p}{d}\wedge \frac{1}{2})}\right)$
Fundamental Limit	$\tilde{\Omega}\left(n^{-(\frac{p}{d}\wedge 1)}\right)$	$\tilde{\Omega}\left(n^{-(\frac{p}{d}\wedge \frac{1}{2})}\right)$

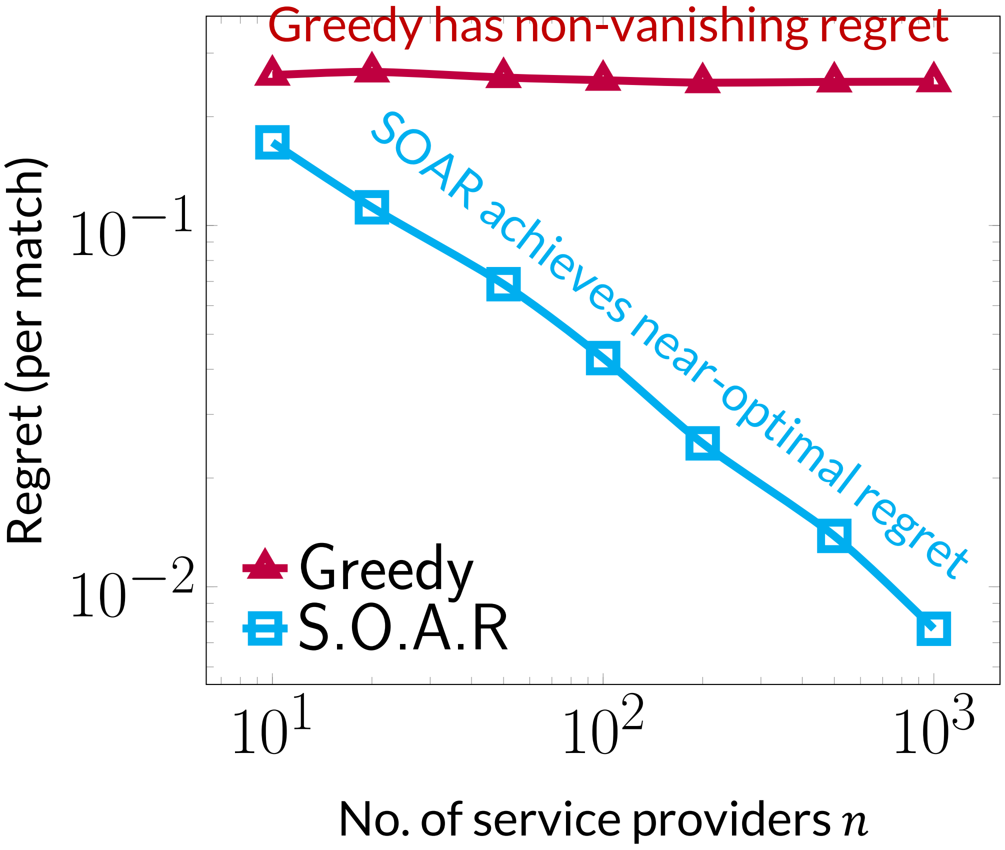
- Resolves one of the open problems in Kanoria (2022)

Summary



Greedy has non-vanishing regret
Regret (per match)
10^{-1}
10^{-2}
Greedy
10^{1}
10^{2}
10^{3}
No. of service providers n

Summary



Greedy has non-vanishing regret
SOAR achieves near-optimal regret
Regret (per match)
10^{-1}
10^{-2}
Greedy
S.O.A.R
10^1
10^2
10^3
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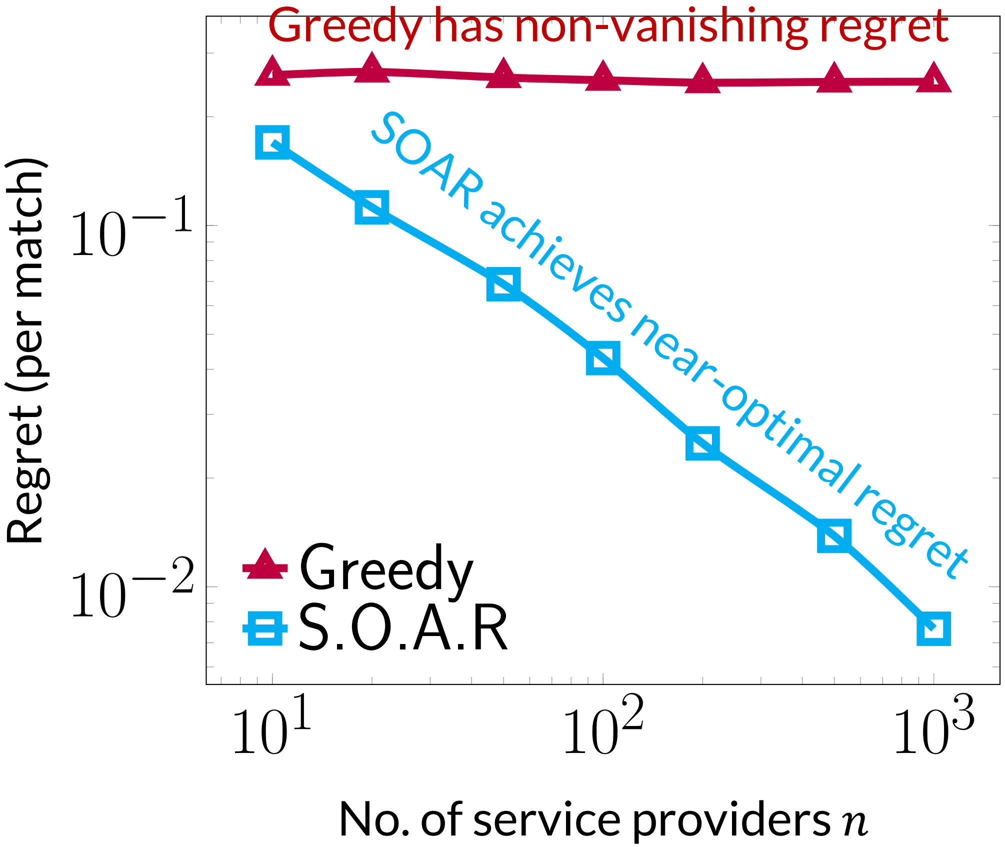
Summary



https://ssrn.com/abstract=4451799

So long, and **Thanks** for all the fish

Appendix

Near-Optimality of SOAR

match quality function $\varphi(X,Y) = \langle X,Y \rangle$

demand distribution P and supply distribution Q

	P, Q are smooth	P, Q are arbitrary
SOAR	$\tilde{O}\left(n^{-\left(\frac{2}{d} \wedge 1\right)}\right)$	$\tilde{O}\left(n^{-\left(\frac{2}{d} \wedge \frac{1}{2}\right)}\right)$
Fundamental Limit	$\tilde{\Omega}\left(n^{-\left(\frac{2}{d} \wedge 1\right)}\right)$	$\tilde{\Omega}\left(n^{-\left(\frac{2}{d} \wedge \frac{1}{2}\right)}\right)$

$(NND)^2$ is a lower bound on regret and $NND \sim n^{-1/d}$ $(\langle X,Y \rangle \equiv -\|X-Y\|^2)$

$d = 1$ matching constraints leads to a tighter lower bound

for arbitrary distributions, a simple example implies that $1/\sqrt{n}$ is a lower bound $(1/\sqrt{n} \gg (NND)^2$ for $d \leq 3)$