Feature Based Dynamic Matching

Yilun Chen
CUHK Shenzhen

Yash Kanoria
Columbia Business School

Akshit Kumar
Columbia Business School

Wenxin Zhang
Columbia Business School

In a Nutshell

We study dynamic centralized matching in two-sided markets with heterogeneous supply and demand

Motivated by applications we assume a spatial structure on the demand and supply type space and resulting matching utility

Myopic policies are highly suboptimal

We design a simple, practical, near-optimal policy **SOAR**

Simulate, Optimize, Assign & Repeat

Two-sided platforms

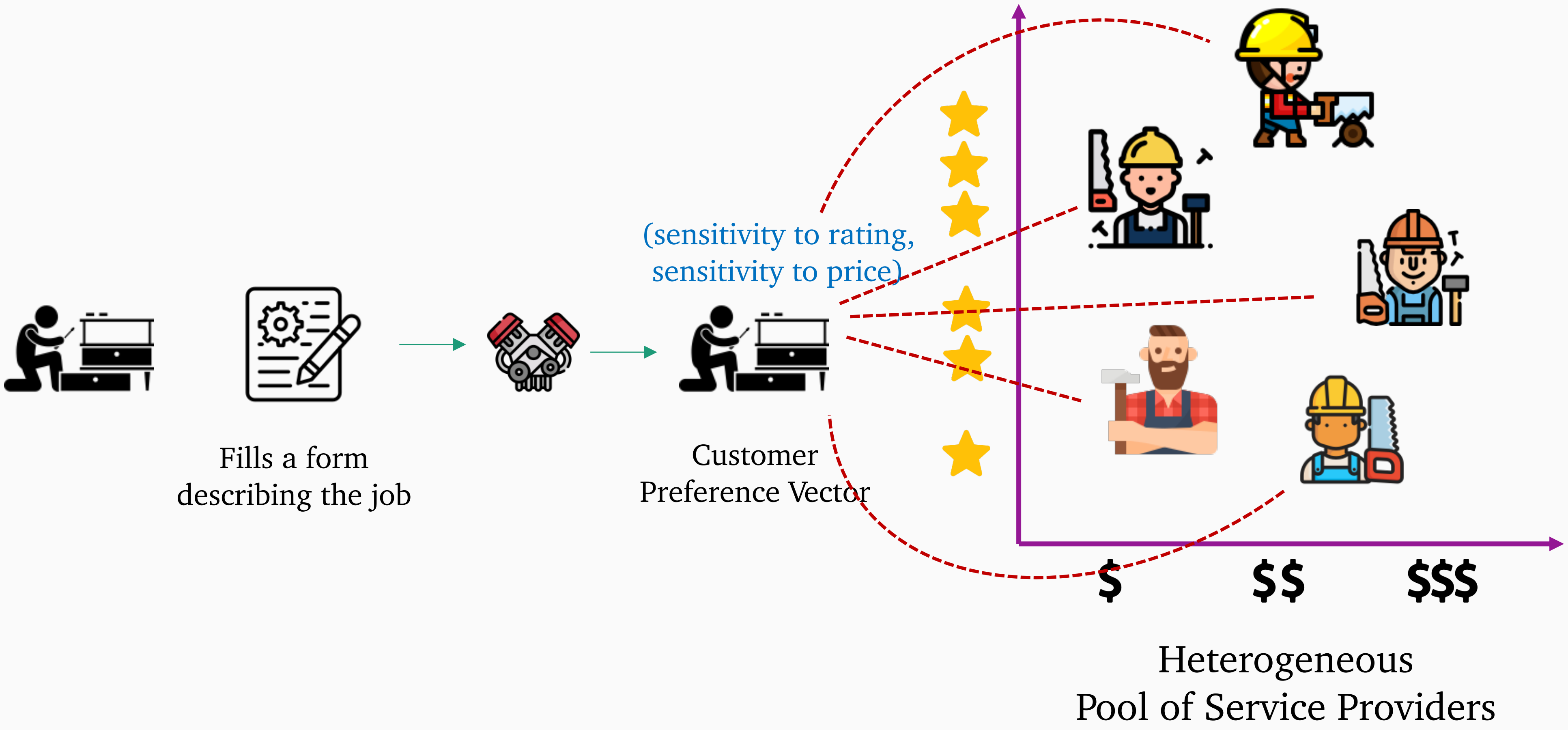
lyft
UBER
via
ARRO
OLA

Angi
handy
taskrabbit
Thumbtack

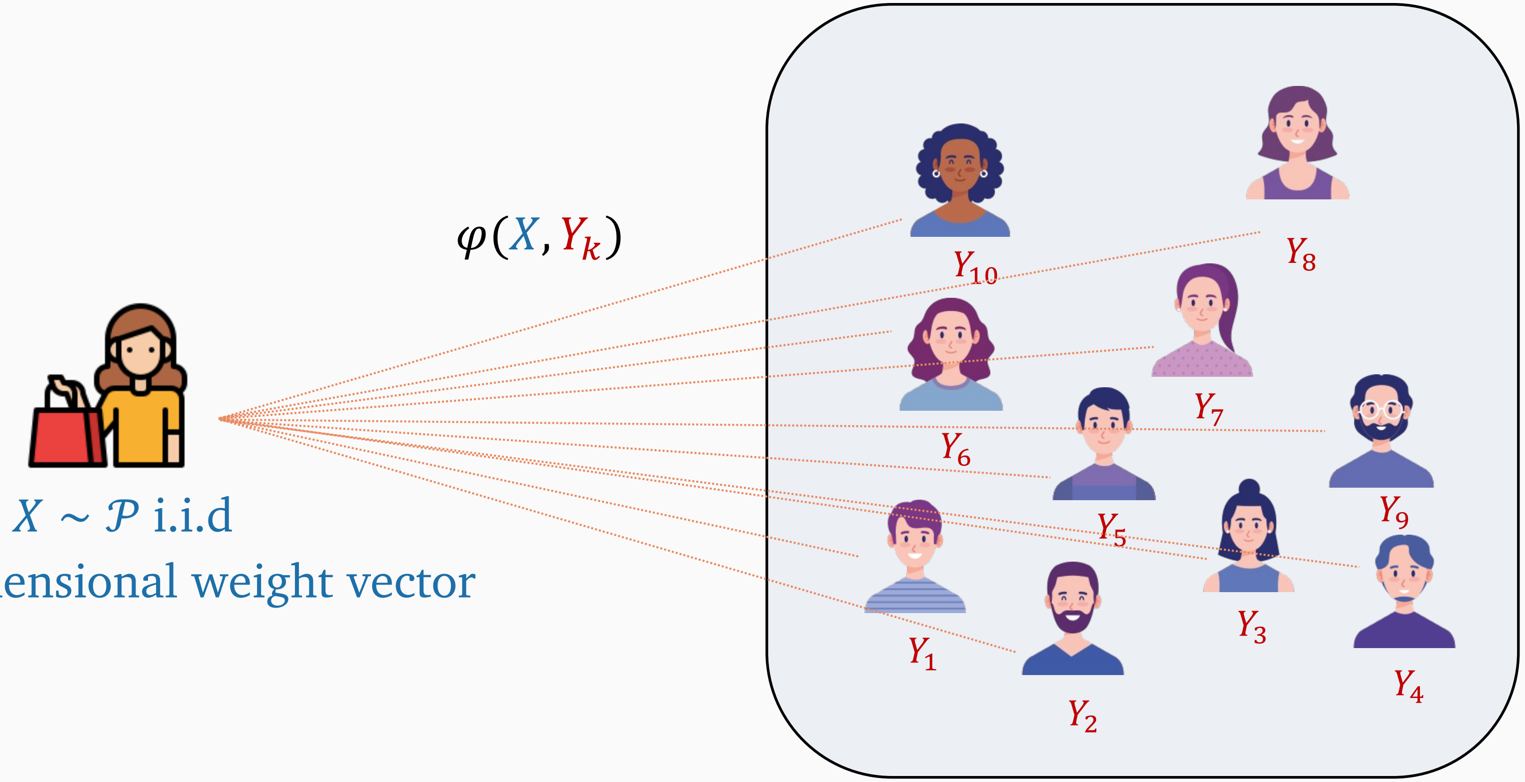
airbnb
agoda
Vrbo
Snaptrip!

Two-sided platforms

Operations


Fills a form
describing the job
(sensitivity to rating,
sensitivity to price)
Customer
Preference Vector
$
$$
$$$
Heterogeneous
Pool of Service Providers

Model



$X \sim \mathcal{P}$ i.i.d
d dimensional weight vector

n service providers
d dimensional feature vector

$\mathcal{P}$ known to the platform
access to historical demand data

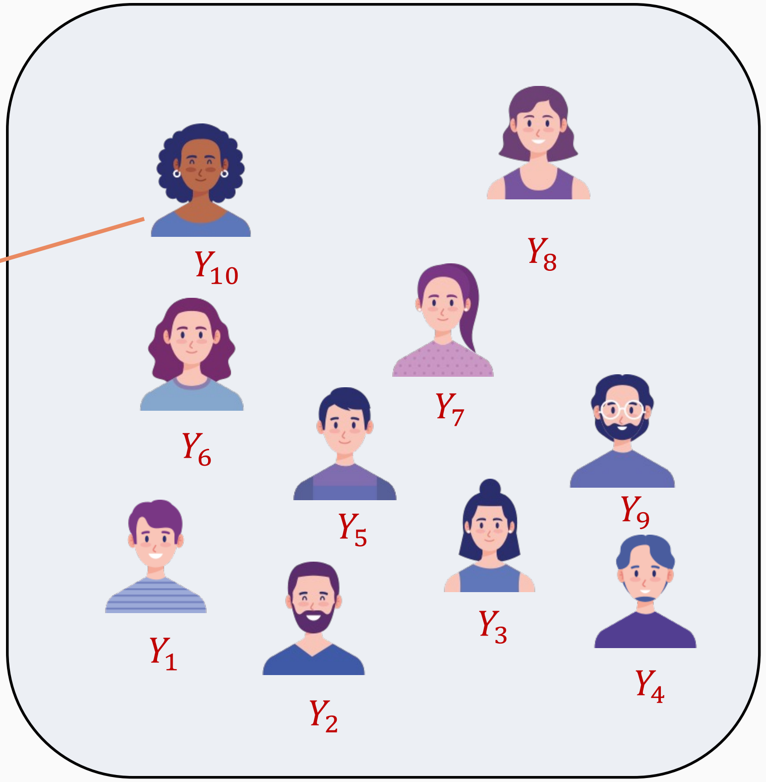
dot product utility $\varphi(X, Y) = \langle X, Y \rangle$

Model

$X \sim \mathcal{P}$ i.i.d

d dimensional weight vector

$\varphi(X, Y_k)$



n service providers
$n - 1$ service providers
d dimensional feature vector

$\mathcal{P}$ known to the platform
access to historical demand data

dot product utility $\varphi(X, Y) = \langle X, Y \rangle$

Related Literature

Dynamic two-sided matching and resource allocation with few types

Talluri & van Ryzin (2006), Vera & Banerjee (2021), Banerjee, Freund & Lykouris (2022)

Online Stochastic Matching with many supply types

Manshadi, Gharan & Saberi (2012)

infinitely many types
spatial structure on type

Static Spatial Matching and Empirical Optimal Transport

Ajtai, Komlos & Tusnady (1984), Talagrand (1992,1994), Shor (1986, 1991), Ledoux (2019), Manole & Niles-Weed (2021)

dynamic demand arrivals

Dynamic Spatial Matching with identical supply and demand distributions

Gupta, Guruganesh, Peng & Wajc (2019), Akbarpour, Alimohammad, Li & Saberi (2022), Kanoria (2022)

different supply & demand distribution

Stochastic Assignment Problems with different supply & demand dists.

Derman, Lieberman & Ross (1972), Su & Zenios (2005), Goldenshluger, Malinovsky & Zeevi (2020)

high dimensional features

Performance Metric

- We aim to maximize the expected average match value $\frac{1}{n}\sum_{k=1}^{n}\langle X_k , Y_{\pi(k)}\rangle$
- Fluid benchmark is the value of the optimal transport between the demand distribution and the supply distribution
- We aim to minimize the additive regret wrt to the fluid benchmark. We want o(1) regret

- Problem is equivalent to minimizing $\frac{1}{n}\sum_{k=1}^{n} ||X_k - Y_{\pi(k)}||^2$

Fundamental Limits

$\mathcal{P}$ known demand distribution

n supply units drawn i.i.d from $\mathcal{Q}$

dot product utility $\varphi(X, Y) = \langle X, Y \rangle$

	$\mathcal{P}, \mathcal{Q}$ regular	$\mathcal{P}, \mathcal{Q}$ arbitrary
Lower Bound (per match regret)	$\widetilde{\Omega}\left(n^{-(\frac{2}{d} \wedge 1)}\right)$	$\widetilde{\Omega}\left(n^{-(\frac{2}{d} \wedge \frac{1}{2})}\right)$

$(NND)^2$ is a lower bound on regret and $NND \sim n^{-1/d}$

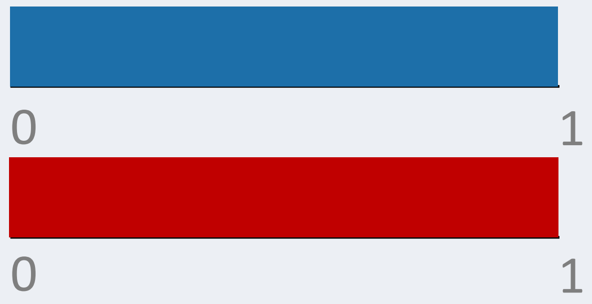
$d = 1$ matching constraints leads to a tighter lower bound

for arbitrary distributions, a simple example implies that $1/\sqrt{n}$ is a lower bound $1/\sqrt{n} \gg (NND)^2$ for $d \leq 3$

What algorithms can achieve these fundamental limits?

Greedy?

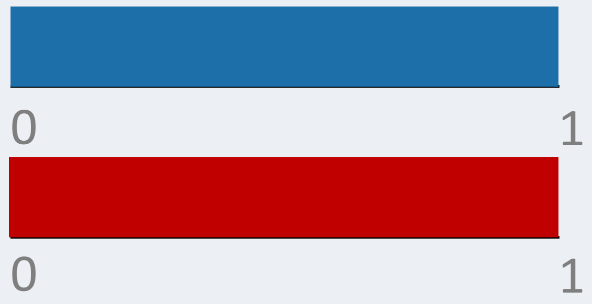
Greedy: The Good, The Bad & The Ugly



$\mathcal{P} = Q$

$\varphi(X, Y) = -||X - Y||$

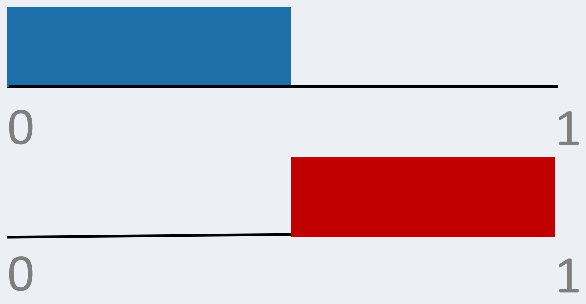
Greedy achieves optimal regret scaling



$\mathcal{P} = Q$

$\varphi(X, Y) = -||X - Y||^2$

Greedy achieves vanishing regret scaling



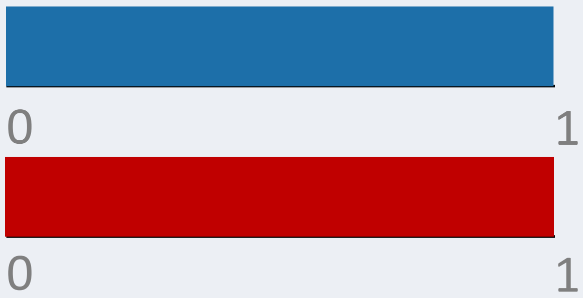
$\mathcal{P} \neq Q$

$\varphi(X, Y) = -||X - Y||^2$

Greedy suffers from **non-vanishing** regret

Greedy is not future aware

Greedy: The Good, The Bad & The Ugly



$\mathcal{P} = Q$

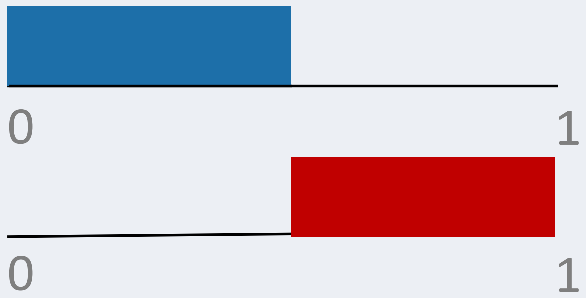
$\varphi(X,Y) = -||X-Y||$

Greedy achieves optimal regret scaling

$\mathcal{P} = Q$

$\varphi(X,Y) = -||X-Y||^2$

Greedy achieves vanishing regret scaling



$\mathcal{P} \neq Q$

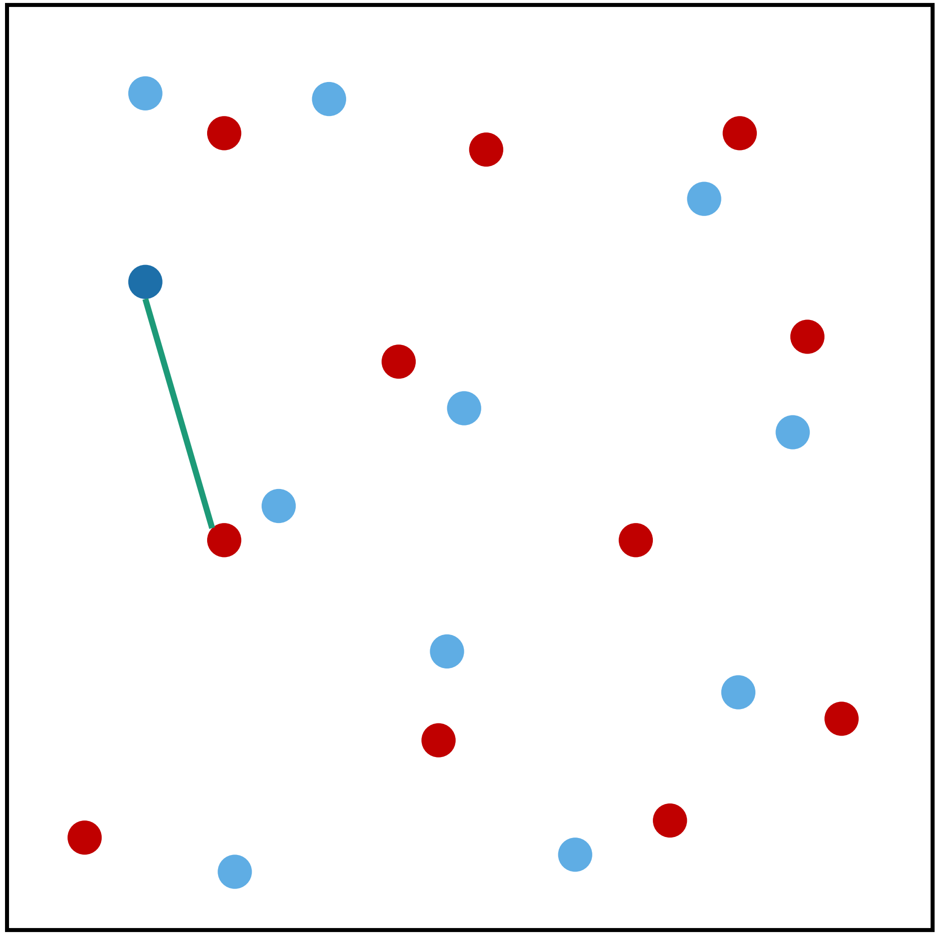
$\varphi(X,Y) = -||X-Y||^2$

Greedy suffers from **non-vanishing** regret

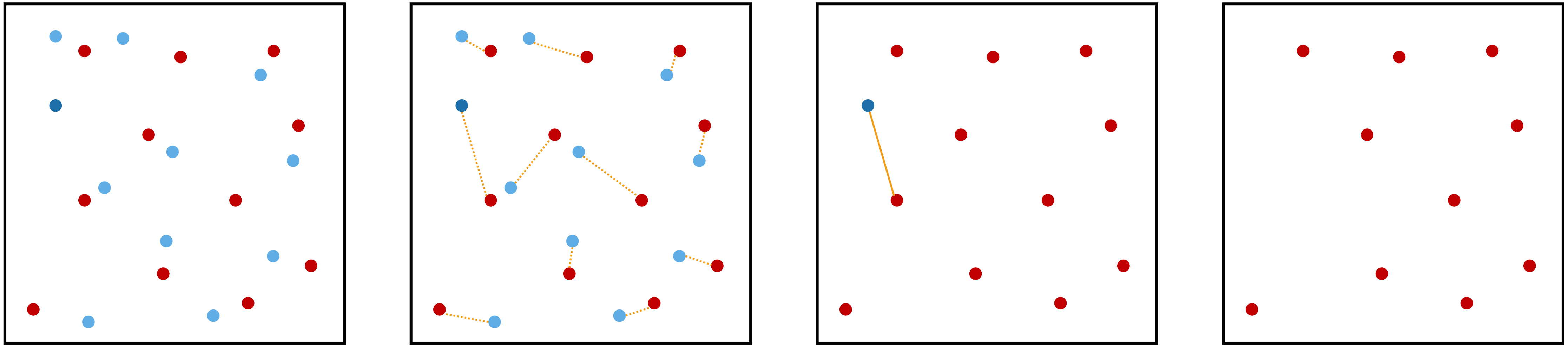
Greedy is not future aware

SOAR: future-aware algorithm

Simulate
Optimize
Assign
Repeat



SOAR: future-aware algorithm



Simulate Optimize Assign Repeat

Fundamental Limits

$\mathcal{P}$ known demand distribution
n supply units drawn i.i.d from $\mathcal{Q}$
dot product utility $\varphi(X, Y) = \langle X, Y \rangle$

	$\mathcal{P}, \mathcal{Q}$ regular	$\mathcal{P}, \mathcal{Q}$ arbitrary
Lower Bound (per match regret)	$\widetilde{\Omega}\left(n^{-\left(\frac{2}{d} \wedge 1\right)}\right)$	$\widetilde{\Omega}\left(n^{-\left(\frac{2}{d} \wedge \frac{1}{2}\right)}\right)$

$(NND)^2$ is a lower bound on regret and $NND \sim n^{-1/d}$
$d = 1$ matching constraints leads to a tighter lower bound
for arbitrary distributions, a simple example implies that $1/\sqrt{n}$ is a lower bound $1/\sqrt{n} \gg (NND)^2$ for $d \leq 3$

SOAR is provably near optimal

$\mathcal{P}$ known demand distribution

n supply units drawn i.i.d from $\mathcal{Q}$

dot product utility $\varphi(X, Y) = \langle X, Y \rangle$

	$\mathcal{P}, \mathcal{Q}$ regular	$\mathcal{P}, \mathcal{Q}$ arbitrary
Lower Bound (per match regret)	$\tilde{\Omega}\left(n^{-\left(\frac{2}{d} \wedge 1\right)}\right)$	$\tilde{\Omega}\left(n^{-\left(\frac{2}{d} \wedge \frac{1}{2}\right)}\right)$
SOAR	$\tilde{O}\left(n^{-\left(\frac{2}{d} \wedge 1\right)}\right)$	$\tilde{O}\left(n^{-\left(\frac{2}{d} \wedge \frac{1}{2}\right)}\right)$

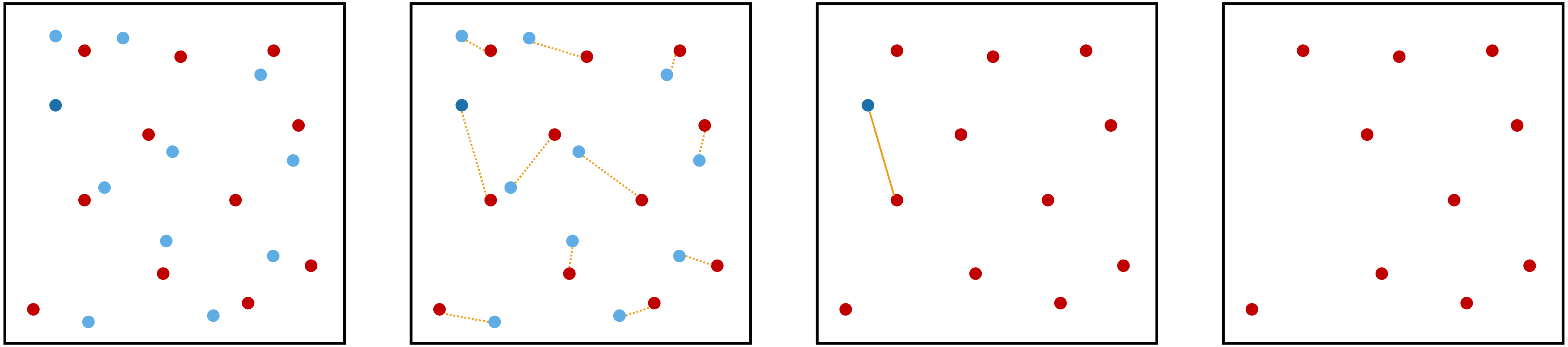
$(NND)^2$ is a lower bound on regret and $NND \sim n^{-1/d}$

$d = 1$ matching constraints leads to a tighter lower bound

for arbitrary distributions, a simple example implies that $1/\sqrt{n}$ is a lower bound $1/\sqrt{n} \gg (NND)^2$ for $d \leq 3$

Key Technical Ideas

k i.i.d supply units — $k-1$ i.i.d supply units



Simulate **O**ptimize **A**ssign **R**epeat

each supply unit is equally likely to be matched

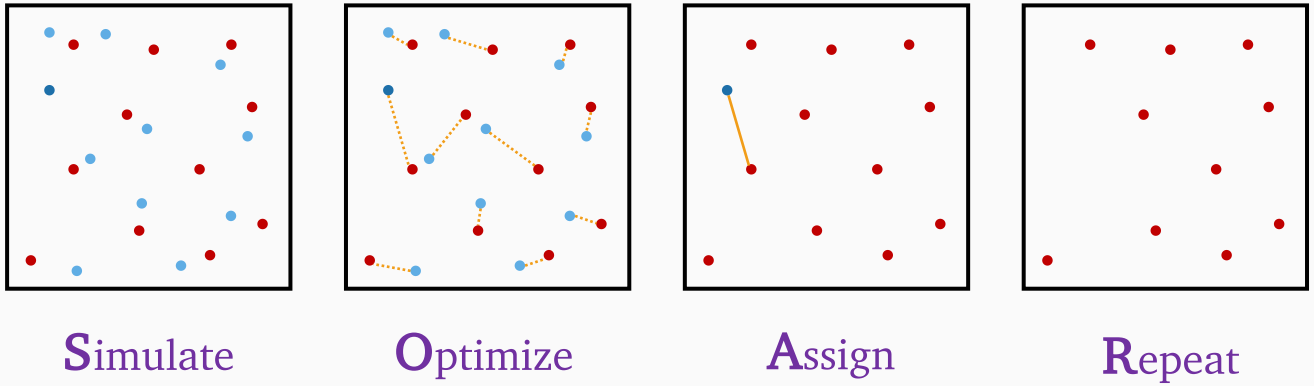
expected matching cost is the average expected matching cost under offline matching

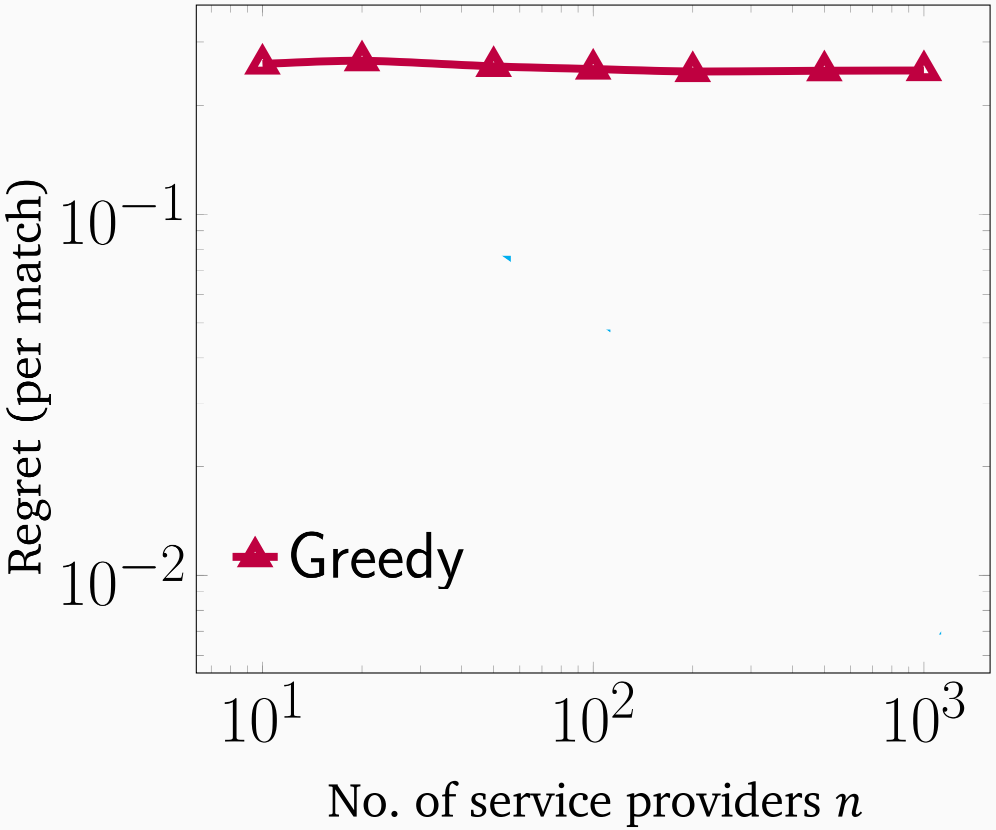
Summary

We study dynamic centralized matching in two-sided markets with heterogeneous supply and demand

Greedy policy suffers from non-vanishing regret

SOAR is a simple, practical and near-optimal policy






SO LONG AND
THANKS FOR
ALL THE FISH